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| January 20, 2014 | Matt Landreman |

# Equations for a minimal stellarator drift-kinetic code for obtaining monoenergetic coefficients

Let’s try to solve the kinetic equation



where I will consider only  for simplicity. In ,  represents a “source” that is independent of all 3 independent variables . As in Beidler 2011, let’s take the collision operator to be

.

We will use Boozer coordinates, so

,

where  is the rotational transform with  the safety factor, and

.

where , ,  is the poloidal current outside the flux surface, and  is the toroidal current inside the flux surface. The product of with gives

.

From the documentation for multispecies SFINCS, we know



and

.

Then becomes



For normalizations, we introduce reference values of magnetic field and length,  and . We introduce normalized quantities







We normalize everything in the usual way, except for the normalization of . We multiply through by



and define the normalized collisionality

.

We let the normalized source be

.

Then becomes



We then define 2 normalized versions of , one for each gradient drive term:

,

,

where  and  are the solutions of





## Legendre modal discretization

We employ the following modal expansion in terms of Legendre polynomials :

.

We discretize the kinetic equations - by applying

.

To evaluate the various integrals that result, the following identities may be used:

,

,

,

,

and

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As a result, we get



and



where



,



## Diagnostics

For computing outputs, we use the following Legendre identities:





One quantity we care about is the radial particle flux:

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Using , we find

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Another quantity we care about is the parallel flow:



where the flux surface average of any quantity  is

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## Conversion to sfincs version 3



Using ,



Using ,

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Hakan’s prescription is to set  and xWeights=exp(1), so the integration weight cancels the Maxwelllian. Thus,

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Using ,

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Using and ,



The  integral gives

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Using Hakan’s convention in which x=1 and xWeights = exp(1),

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For comparison, the particle flux in sfincs version 3 is

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## Summary of equations:

In this section I will drop decorations wherever possible. Before the Legendre modal expansion, the kinetic equation is

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After the Legendre expansion, the kinetic equation becomes



where



and

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